WEEKLY TEST TARGET JEE- -02 MATHEMATICS SOLUTION 13 JULY 2019

MATHEMATICS

61. (c) Given, diameter of circular wire = $10cm$, therefore length of wire = 10π.

Hence required angle $= \frac{\text{arc}}{\text{radius}} = \frac{10\pi}{50} = \frac{\pi}{5}$ *radian*.

62. (d) Given expression is

$\sin^2 5^o + \sin^2 10^o + \sin^2 15^o + ... + \sin^2 85^o + \sin^2 90^o$.

We know that $\sin 90^o = 1$ or $\sin^2 90^o = 1$.

Similarly, $\sin 45^o = \frac{1}{\sqrt{2}}$ or $\sin^2 45^o = \frac{1}{2}$ and the angles are in A.P. of 18 terms. We also know that

$\sin^2 85^o = [\sin(90^o - 5^o)]^2 = \cos^2 5^o$.

Therefore from the complementary rule, we find $\sin^2 5^o + \sin^2 85^o = \sin^2 5^o + \cos^2 5^o = 1$.
Therefore,

$\sin^2 5^o + \sin^2 10^o + \sin^2 15^o + ... + \sin^2 85^o + \sin^2 90^o$

$= (1+1+1+1+1+1+1+1) + 1 + \frac{1}{2} = 9\frac{1}{2}$.

63. (c) $\sqrt{\text{cosec}^2\alpha + 2\cot\alpha} = \sqrt{1 + \cot^2\alpha + 2\cot\alpha} = |1 + \cot\alpha|$

But $\frac{3\pi}{4} < \alpha < \pi \Rightarrow \cot\alpha < -1 \Rightarrow 1 + \cot\alpha < 0$

Hence, $|1 + \cot\alpha| = -(1 + \cot\alpha)$.

64. (b) We have $x + \frac{1}{x} = 2\cos\theta$,

Now $x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3x\frac{1}{x}\left(x + \frac{1}{x}\right)$

$= (2\cos\theta)^3 - 3(2\cos\theta) = 8\cos^3\theta - 6\cos\theta$

$= 2(4\cos^3\theta - 3\cos\theta) = 2\cos 3\theta$.

Trick : Put $x = 1 \Rightarrow \theta = 0^\circ$.

Then $x^3 + \frac{1}{x^3} = 2 = 2\cos 3\theta$.

65. (a) $\sin x + \text{cosec} x = 2 \Rightarrow (\sin x - 1)^2 = 0 \Rightarrow \sin x = 1$

$\therefore \sin^n x + \text{cosec}^n x = 1 + 1 = 2$.

66. (b) Since $\cos^6\alpha + \sin^6\alpha + K\sin^2 2\alpha = 1$ using formula $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$ and on solving, we get the required result *i.e.* $K = \frac{3}{4}$.

67. (c) $\sin 20^\circ \sin 40^o \sin 60^\circ \sin 80^\circ = \frac{1}{2}\sin 20^\circ \sin 60^\circ (2\sin 40^o \sin 80^\circ)$

$= \frac{1}{2}\sin 20^\circ \sin 60^\circ(\cos 40^\circ - \cos 120^\circ)$

$= \frac{1}{2}.\frac{\sqrt{3}}{2}\sin 20^\circ\left(1 - 2\sin^2 20^\circ + \frac{1}{2}\right)$

$= \frac{\sqrt{3}}{4}\sin 20^\circ\left(\frac{3}{2} - 2\sin^2 20^\circ\right)$

$$= \frac{\sqrt{3}}{8}(3\sin 20^\circ - 4\sin^3 20^\circ) \qquad = \frac{\sqrt{3}}{8}\sin 60^\circ = \frac{\sqrt{3}}{8}.\frac{\sqrt{3}}{2} = \frac{3}{16}.$$

68. (d) $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14}$

$$= \sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\times 1$$

$$\times \sin\left(\pi - \frac{5\pi}{14}\right)\sin\left(\pi - \frac{3\pi}{14}\right)\sin\left(\pi - \frac{\pi}{14}\right)$$

$$= \left[\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}\right]^2 = \frac{1}{64}.$$

69. (b) $x = \sin 130^o \cos 80^o$, $y = \sin 80^o \cos 130^o$

$\Rightarrow x = \cos 40^o \cos 80^o$, $y = -\sin 80^o \sin 40^o$

So, $x > 0$ and $y < 0$ and $xy < 0$

Now, $z = 1 + xy \Rightarrow 0 < z < 1$.

70. (b) $a\cos 2\theta + b\sin 2\theta = c$

$$\Rightarrow a\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) + b\frac{2\tan\theta}{1+\tan^2\theta} = c$$

$$\Rightarrow a - a\tan^2\theta + 2b\tan\theta = c + c\tan^2\theta$$

$$\Rightarrow -(a+c)\tan^2\theta + 2b\tan\theta + (a-c) = 0$$

$$\therefore \tan\alpha + \tan\beta = -\frac{2b}{-(c+a)} = \frac{2b}{c+a}$$

71. (b) Given, $\operatorname{cosec}\theta = \frac{p+q}{p-q} \Rightarrow \frac{1}{\sin\theta} = \frac{p+q}{p-q}$

Apply componendo and dividendo

$$\frac{1+\sin\theta}{1-\sin\theta} = \frac{p+q+p-q}{p+q-p+q}$$

$$\Rightarrow \left\{\frac{\cos\frac{\theta}{2} + \sin\frac{\theta}{2}}{\cos\frac{\theta}{2} - \sin\frac{\theta}{2}}\right\}^2 = \frac{p}{q} \Rightarrow \left\{\frac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}}\right\}^2 = \frac{p}{q}$$

$$\Rightarrow \tan^2\left(\frac{\pi}{4} + \frac{\theta}{2}\right) = \frac{p}{q} \Rightarrow \cot^2\left(\frac{\pi}{4} + \frac{\theta}{2}\right) = \frac{q}{p}$$

Note : $\cot\left(\frac{\pi}{4} + \frac{\theta}{2}\right) = \sqrt{\frac{q}{p}}$ only, if $\cot\left(\frac{\pi}{4} + \frac{\theta}{2}\right) > 0$.

72. (b, c) The expression reduces to $\cot^n\frac{A-B}{2} + \cot^n\frac{B-A}{2}$

If n is even, answer is (b) and if n is odd answer is (c).

73. (a, c) We have $\sin\alpha = 1/\sqrt{5} \Rightarrow \cos\alpha = 2/\sqrt{5}$

and $\sin\beta = 3/5 \Rightarrow \cos\beta = 4/5$

$$\sin(\beta - \alpha) = \sin\beta\cos\alpha - \sin\alpha\cos\beta$$

$$= \frac{3}{5}.\frac{2}{\sqrt{5}} - \frac{1}{\sqrt{5}}.\frac{4}{5} = \frac{2}{5\sqrt{5}} = 0.1789$$

Now $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} = 0.7071 = \sin\frac{3\pi}{4}$

Since $0 < 0.1789 < 0.7071$

$$\therefore \sin 0 < \sin(\beta - \alpha) < \sin\frac{\pi}{4} \Rightarrow 0 < (\beta - \alpha) < \frac{\pi}{4}$$

Also, $\sin\pi < \sin(\beta - \alpha) < \sin\frac{3\pi}{4}$

$\therefore (\beta - \alpha) \in [0, \pi/4]$ and $[3\pi/4, \pi]$.

74. (a) The given equation may be written as

$$\frac{2}{\cos 2\alpha}=\frac{\sin\beta}{\cos\beta}+\frac{\cos\beta}{\sin\beta}=\frac{\sin^2\beta+\cos^2\beta}{\cos\beta\sin\beta}=\frac{1}{\cos\beta.\sin\beta}$$

$\Rightarrow \cos 2\alpha=\sin 2\beta$

$\Rightarrow \cos 2\alpha = \cos\left(\frac{\pi}{2}-2\beta\right)\Rightarrow 2\alpha=\frac{\pi}{2}-2\beta$

$\Rightarrow 2\alpha+2\beta=\frac{\pi}{2}\Rightarrow \alpha+\beta=\frac{\pi}{4}$.

75. (b) We have $\frac{x}{\cos\theta}=\frac{y}{\cos\left(\theta-\frac{2\pi}{3}\right)}=\frac{z}{\cos\left(\theta+\frac{2\pi}{3}\right)}=k$

$\Rightarrow x=k\cos\theta$, $y=k\cos\left(\theta-\frac{2\pi}{3}\right)$, $z=k\cos\left(\theta+\frac{2\pi}{3}\right)$

$\Rightarrow x+y+z=k\left[\cos\theta+\cos\left(\theta-\frac{2\pi}{3}\right)+\cos\left(\theta+\frac{2\pi}{3}\right)\right]$

$=k[(0)=0$

$\Rightarrow x+y+z=0$.

76. (d) $\sin 6\theta=2\sin 3\theta\cos 3\theta$

$=2[3\sin\theta-4\sin^3\theta][4\cos^3\theta-3\cos\theta]$

$=24\sin\theta\cos\theta(\sin^2\theta+\cos^2\theta)-18\sin\theta\cos\theta-32\sin^2\theta\cos^2\theta$

$=32\cos^5\theta\sin\theta-32\cos^3\theta\sin\theta+3\sin 2\theta$

On comparing, $x=\sin 2\theta$.

77. (c) $\sin^4\frac{\pi}{8}+\sin^4\frac{3\pi}{8}+\sin^4\frac{5\pi}{8}+\sin^4\frac{7\pi}{8}$

$=\frac{1}{4}\left[\left(2\sin^2\frac{\pi}{8}\right)^2+\left(2\sin^2\frac{3\pi}{8}\right)^2\right]$

$+\frac{1}{4}\left[\left(2\sin^2\frac{\pi}{8}\right)^2+\left(2\sin^2\frac{3\pi}{8}\right)^2\right]$

$=\frac{1}{4}\left[\left(1-\cos\frac{\pi}{4}\right)^2+\left(1-\cos\frac{3\pi}{4}\right)^2\right]$

$+\frac{1}{4}\left[\left(1-\cos\frac{\pi}{4}\right)^2+\left(1-\cos\frac{3\pi}{4}\right)^2\right]$

$=\frac{1}{4}\left[\left(1-\frac{1}{\sqrt{2}}\right)^2+\left(1+\frac{1}{\sqrt{2}}\right)^2\right]+\frac{1}{4}\left[\left(1-\frac{1}{\sqrt{2}}\right)^2+\left(1+\frac{1}{\sqrt{2}}\right)^2\right]$

$=\frac{1}{4}(3)+\frac{1}{4}(3)=\frac{3}{2}$.

78. (a) We have $\cot A=\frac{\cos A}{\sin A}=\frac{2\cos^2 A}{2\sin A\cos A}=\frac{1+\cos 2A}{\sin 2A}$

Putting $A=7\frac{1^o}{2}\Rightarrow \cot 7\frac{1^o}{2}=\frac{1+\cos 15^o}{\sin 15^o}$

On simplification,

we get $\cot 7\frac{1^o}{2}=\sqrt{6}+\sqrt{2}+\sqrt{3}+\sqrt{4}$.

79. (b) Let $f(x)=\sin\theta+\cos\theta=\sqrt{2}\sin\left(\theta+\frac{\pi}{4}\right)$

But $-1\le\sin\left(\theta+\frac{\pi}{2}\right)\le 1\Rightarrow -\sqrt{2}\le\sqrt{2}\sin\left(\theta+\frac{\pi}{4}\right)\le\sqrt{2}$.

Hence the maximum value of $(\sin\theta+\cos\theta)$

i.e., of $\sqrt{2}\sin\left(\theta+\frac{\pi}{4}\right)=\sqrt{2}$.

$\therefore \sin\left(\theta+\frac{\pi}{4}\right)=1\Rightarrow \sin\left(\theta+\frac{\pi}{4}\right)=\sin\frac{\pi}{2}$

$\Rightarrow \theta + \frac{\pi}{4} = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{4} = 45^{o}$.

80. (a) Let $y = \frac{\tan x}{\tan 3x} = \frac{\tan x}{\frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}}$

$y = \frac{1 - 3\tan^2 x}{3 - \tan^2 x} = \frac{\frac{1}{3} - \tan^2 x}{1 - \frac{1}{3}.\tan^2 x}$

Hence, y should never lie between $\frac{1}{3}$ and 3 whenever defined.

81. (c) We have $\cos 2\theta + 2\cos\theta = 2\cos^2\theta - 1 + 2\cos\theta$

$$= 2\left(\cos\theta + \frac{1}{2}\right)^2 - \frac{3}{2}$$

Now $2\left(\cos\theta + \frac{1}{2}\right)^2 \geq 0$ for all θ

$\therefore 2\left(\cos\theta + \frac{1}{2}\right)^2 - \frac{3}{2} \geq \frac{-3}{2}$ for all θ .

$\Rightarrow \cos 2\theta + 2\cos\theta \geq \frac{-3}{2}$ for all θ

Also max. value of this expression is 3.

82. (b) $\cos[\pi - (B + C)] = \cos B \cos C$

$\Rightarrow -\cos(B + C) = \cos B \cos C$

$\Rightarrow -[\cos B \cos C - \sin B \sin C] = \cos B \cos C$

$\Rightarrow \sin B \sin C = 2\cos B \cos C$

$\Rightarrow \tan B \tan C = 2$.

83. (b) Let $u = \cos\theta\left\{\sin\theta + \sqrt{\sin^2\theta + \sin^2\alpha}\right\}$

$\Rightarrow (u - \sin\theta\cos\theta)^2 = \cos^2\theta(\sin^2\theta + \sin^2\alpha)$

$\Rightarrow u^2\tan^2\theta - 2u\tan\theta + u^2 - \sin^2\alpha = 0$

Since tan θ is real, therefore

$\Rightarrow 4u^2 - 4u^2(u^2 - \sin^2\alpha) \geq 0$

$\Rightarrow u^2 - (1 + \sin^2\alpha) \leq 0$

$\Rightarrow |u| \leq \sqrt{1 + \sin^2\alpha}$.

84. (b) $\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$

$\Rightarrow 2\sin 2x\cos x - 3\sin 2x - 2\cos 2x\cos x + 3\cos 2x = 0$

$\Rightarrow \sin 2x(2\cos x - 3) - \cos 2x(2\cos x - 3) = 0$

$\Rightarrow (\sin 2x - \cos 2x)\ (2\cos x - 3) = 0 \Rightarrow \sin 2x = \cos 2x$

$\Rightarrow 2x = 2n\pi \pm \left(\frac{\pi}{2} - 2x\right)$ *i.e.*, $x = \frac{n\pi}{2} + \frac{\pi}{8}$.

85. (b) $5 - 5\sin^2\theta + 7\sin^2\theta = 6 \Rightarrow 2\sin^2\theta = 1$

$\Rightarrow \sin^2\theta = \frac{1}{2} = \sin^2\left(\frac{\pi}{4}\right) \Rightarrow \theta = n\pi \pm \frac{\pi}{4}$.

86. (b) $\tan\theta = \frac{-1}{\sqrt{3}} = \tan\left(\pi - \frac{\pi}{6}\right)$, $\sin\theta = \frac{1}{2} = \sin\left(\pi - \frac{\pi}{6}\right)$ and $\cos\theta = \frac{-\sqrt{3}}{2} = \cos\left(\pi - \frac{\pi}{6}\right)$

Hence principal value is $\theta = \frac{5\pi}{6}$.

87. (c) $\sec 4\theta - \sec 2\theta = 2$

$\Rightarrow \cos 2\theta - \cos 4\theta = 2\cos 4\theta \cos 2\theta$

$\Rightarrow -\cos 4\theta = \cos 6\theta \Rightarrow 2\cos 5\theta \cos\theta = 0$

$\Rightarrow \theta = n\pi + \frac{\pi}{2}$ or $\frac{n\pi}{5} + \frac{\pi}{10}$.

88. (a) $2\sin 3x \cos x - 2\sin 3x = 0$, $\therefore \sin 3x = 0$, $\cos x = 1$

$\Rightarrow 3x = n\pi$ or $x = \frac{n\pi}{3}$ and $x = 2n\pi$

The second value $x = 2n\pi$ is included in the value given by $x = \frac{n\pi}{3}$.

89. (b) $\tan(\cot x) = \cot(\tan x) \Rightarrow \tan(\cot x) = \tan\left(\frac{\pi}{2} - \tan x\right)$

$\Rightarrow \cot x = n\pi + \frac{\pi}{2} - \tan x \Rightarrow \cot x + \tan x = n\pi + \frac{\pi}{2}$

$\Rightarrow \frac{2}{\sin 2x} = n\pi + \frac{\pi}{2} \Rightarrow \sin 2x = \frac{2}{n\pi + \frac{\pi}{2}} = \frac{4}{(2n+1)\pi}$.

90. (c) $(5 + 4\cos\theta)(2\cos\theta + 1) = 0$

$\cos\theta = -5/4$, which is not possible.

$\therefore 2\cos\theta + 1 = 0$ or $\cos\theta = -1/2$

$\Rightarrow \theta = \frac{2\pi}{3}, \frac{4\pi}{3}$. Solution set is $\left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\} \in [0, 2\pi]$.